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#### Introduction

Johannes Åman Pohjola University of New South Wales Term 2 2023



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# Meet the staff

I am Johannes Åman Pohjola. I'm a lecturer at UNSW. I work on the applications of formal mathematical methods to the development of safe and secure software. He knows nothing about beer.

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# Meet the staff

I am Johannes Åman Pohjola. I'm a lecturer at UNSW. I work on the applications of formal mathematical methods to the development of safe and secure software. He knows nothing about beer.

Scott Buckley will deliver most of the practical lectures (Thursday). He is a postdoctoral researcher, working on formal mathematical methods for reasoning about timing side-channels. He knows everything about beer.

Tsun Wang Sau is the course admin.

Rahul Tripathi and Raphael Douglas Giles will help out with assignments, forum interaction and more. Rahul knows some things about beer.

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# **Contacting Us**

http://www.cse.unsw.edu.au/~cs3141

#### Forum

There is an Ed forum linked on the course website. Ask questions there. To avoid spoiling solutions, you can and should ask private questions.

Administrative questions should be sent to the course email

cs3141@cse.unsw.edu.au

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# **Student Support**

#### For help with anything else, there is always

Student Support - I N	eed Help With	Screenshot This Slide
Uni and Life in Australia Stress, Financial, Visas, Accommodation & More	🙀 Student Support	student.unsw.edu.au/advisors
Reporting Sexual Assault/Harassment	Equity Diversity and Inclusion (EDI)	edi.unsw.edu.au/ <b>sexual-misconduct</b>
Educational Adjustments To Managemy Studies and Disability / Health Condition	Equitable Learning Services (ELS)	student.unsw.edu.au <b>/els</b>
Academic and Study Skills	🐔 Academic Skills	student.unsw.edu.au <b>/skills</b>
Special Consideration Because Life Impacts our Studies and Exams	Special Consideration	student.unsw.edu.au/special-consideration
My Feelings and Mental Health Managing Low Mood, Unusual Feelings & Depression	Mental Health Connect Student.unsw.edu.au/counsell Telehealth	In Australia Call Afterhours 1300 787 026 UNSW Mental Health Support Line 5pm-9am
	Mind HUB student.unsw.edu.au/mind-hu Online Self-Help Resources	b Outside Australia Afterhours +61 (2) 89050307

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# What is this course?

# Our software should be correct, safe and secure.

# Our software should be developed cheaply and quickly.



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# **Safety-uncritical Applications**



Video games: Some bugs are acceptable, to save developer effort.

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# **Safety-critical Applications**

Think of the worst group assignment you ever had!

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# Safety-critical Applications

Think of the worst group assignment you ever had! Imagine you...

- are logging into your online banking...
- are investing in a new hedge fund...
- are travelling in a self-driving car...
- are travelling on a plane...
- are getting treatment from a radiation therapy machine...
- are about to launch nuclear missiles...

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# Safety-critical Applications

Think of the worst group assignment you ever had! Imagine you...

- are logging into your online banking...
- are investing in a new hedge fund...
- are travelling in a self-driving car...
- are travelling on a plane...
- are getting treatment from a radiation therapy machine...
- are about to launch nuclear missiles...
- ... using software written by your groupmates from that group.

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# **Safety-critical Applications**

# Airline Blames Bad Software in San Francisco Crash Ehe New York Eimes



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## What is this course?

# Maths COMP3141 Software

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# What is this course?



MATH1081 is neither necessary nor sufficient for COMP3141.

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# What is this course?



N.B: Haskell knowledge is not a prerequisite for COMP3141.

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# What this course is not?

What this course is not?

• not a Haskell course

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### What this course is not?

- not a Haskell course
- not a formal verification course (see COMP4161),

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- not a Haskell course
- not a formal verification course (see COMP4161),
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- not a programming languages course (see COMP3161).

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# What this course is not?

- not a Haskell course
- not a formal verification course (see COMP4161),
- not an OOP software design course (see COMP2511),
- **not** a programming languages course (see COMP3161).
- Certainly **not** a cakewalk; but hopefully **not** a soul-crushing nightmare either.

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### Assessment

#### Warning

For many of you, this course will present a lot of new topics. Even if you are a seasoned programmer, you may have to learn as if you were starting from scratch.

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### Assessment

#### Warning

For many of you, this course will present a lot of new topics. Even if you are a seasoned programmer, you may have to learn as if you were starting from scratch.

- Class Marks (out of 100)
  - Two programming assignments, each worth 20 marks.
  - Weekly online quizzes, worth 20 marks.
  - Weekly programming exercises, worth 40 marks.
- Final Exam Marks (out of 100, hurdle: 40)

$$\mathit{result} = \frac{\mathit{class} + \mathit{exam}}{2}$$

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### Lectures

- Lecture (Wed 1pm-3pm): I introduce new material.
- **Practical (Thu 11am-1pm):** Scott (usually) reinforces Wednesday's material with questions and examples.
- **Quiz**: due on Thu (one week after the lectures they examine), but start early!

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#### We won't set a textbook (a long COMP3141 tradition).

**Resources**: see the course outline for various books and online resources that are useful for learning Haskell.

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# Why Haskell?

• This course uses Haskell, because it is the most widely used language with good support for *mathematically structured programming*.

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# Why Haskell?

- This course uses Haskell, because it is the most widely used language with good support for *mathematically structured programming*.
- You will learn a substantial amount of Haskell (we will provide some guidance). But the course is about learning techniques for mathematically structured programming.

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## **About Haskell**

• Haskell is old!



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### **About Haskell**

• Haskell is old! It's turning 33 this year.

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### About Haskell

- Haskell is old! It's turning 33 this year.
- Throughout the years: Haskell 98, Haskell 2010, GHC2021.

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### About Haskell

- Haskell is old! It's turning 33 this year.
- Throughout the years: Haskell 98, Haskell 2010, GHC2021.

#### Warning

This means that some (possibly even most) tutorials, resources, answers you find on the Internet will be outdated!

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### **Demo 1: Haskell Workflow**

- Now we'll give you a Haskell Crash Course.
- This is to get you coding (solving problems) quickly.

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# **Demo 1: Haskell Workflow**

- Now we'll give you a Haskell Crash Course.
- This is to get you coding (solving problems) quickly.
- If you prefer "deep" understanding, don't worry: next week.

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# **Demo 1: Haskell Workflow**

- Now we'll give you a Haskell Crash Course.
- This is to get you coding (solving problems) quickly.
- If you prefer "deep" understanding, don't worry: next week. Demo: GHCi. Modules

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### **Demo 2: Declaring Functions**

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## **Demo 2: Declaring Functions**

In mathematics, we would apply a function f to an argument x by writing f(x). In Haskell we write f x, omitting the parentheses.

#### **Demo: basic functions**

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# Demo 3: Currying

• Haskell functions have one input domain and one output codomain. But some functions take multiple inputs.

```
Overview
```

# Demo 3: Currying

- Haskell functions have one input domain and one output codomain. But some functions take multiple inputs.
- In mathematics, we treat  $\log_{10}(x)$  and  $\log_2(x)$  and  $\ln(x)$  as separate functions.
- In Haskell, we have a single function logBase that, given a number n, produces a function for log<sub>n</sub>(x).

```
log10 :: Double -> Double
log10 = logBase 10
```

```
log2 :: Double -> Double
log2 = logBase 2
```

```
ln :: Double -> Double
ln = logBase 2.71828
What's the type of logBase?
```

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# Demo 3: Currying

#### logBase :: Double -> (Double -> Double)

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# Demo 3: Currying

# logBase :: Double -> (Double -> Double) (parentheses are optional above, we could write:) logBase :: Double -> Double -> Double

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# Demo 3: Currying

# logBase :: Double -> (Double -> Double) (parentheses are optional above, we could write:) logBase :: Double -> Double -> Double

Function application associates to the left in Haskell, so:

logBase 2 64  $\equiv$  (logBase 2) 64

#### Demo: currying, multiple arguments

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## **Demo 4: Tuples**

We now know how to handle multiple inputs to a function? But what if we want to have multiple outputs?

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# **Demo 4: Tuples**

We now know how to handle multiple inputs to a function? But what if we want to have multiple outputs? Haskell provides data types called tuples to handle multiple outputs:

```
neighbors :: Int -> (Int, Int)
neighbors x = (x - 1, x + 1)
```

Now, (neighbors 1) evaluates to (0,2).

#### **Demo: tuples**

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# **Demo 5: Higher Order Functions**

In addition to returning functions, functions can take other functions as arguments:

```
applyTwice :: (t -> t) -> t -> t
applyTwice f x = f (f x)
```

```
square :: Int -> Int
square x = x * x
```

fourthPower :: Int -> Int
fourthPower = applyTwice square
Demo: higher-order functions, equational reasoning

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# Demo 6: Lists

Haskell makes extensive use of lists, constructed using square brackets. Each list element must be of the same type.

[True, False, True]	::	[Bool]
[3, 2, 5+1]	::	[Int]
[sin, cos]	::	[Double -> Double]
[ (3,'a'),(4,'b') ]	::	[(Int, Char)]

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# Demo 6: Lists

A useful function is map, which, given a function, applies it to each element of a list:

map not [True, False, True] = [False, True, False]
map square [3, -2, 4] = [9, 4, 16]
map (\x -> x + 1) [1, 5] = [2, 6]

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# Demo 6: Lists

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The last example here uses a *lambda expression* to define a one-use function without giving it a name.

What's the type of map?

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# Demo 6: Lists

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The last example here uses a *lambda expression* to define a one-use function without giving it a name.

#### What's the type of map?

map :: (a -> b) -> [a] -> [b]

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## Demo 6: Lists

```
The type String in Haskell is just a list of characters:
type String = [Char]
This is a type synonym, like a typedef in C.
```

Thus:

```
"hi!" == ['h', 'i', '!']
```

**Demo: lists** 

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# **Word Frequencies**

Let's solve a problem to get some practice implementing stuff:

#### Example (Task 1)

Given a number n and a string s containing English words, generate a report that lists the n most common words in the given string s.

I'll even give you an algorithm:

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# **Word Frequencies**

Let's solve a problem to get some practice implementing stuff:

#### Example (Task 1)

Given a number n and a string s containing English words, generate a report that lists the n most common words in the given string s.

I'll even give you an algorithm:

- Break the input string into words.
- Onvert the words to lowercase.
- Sort the words.
- Group adjacent occurrences (runs) of the same word.
- Sort runs words by length.
- Take the longest *n* runs of the sorted list.
- Generate a report.

#### Demo: word frequencies

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# The Dollar Pattern

We used *the dollar operator* \$ to reduce the use of parentheses.

- The dollar operator does normal function application, like f x (evaluation of a function at a value).
- However, while application has high operator precedence ("is done as early as possible"), the dollar operator has extremely low precedence ("is done as late as possible").

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# The Dollar Pattern

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- reverse [1,2,3] ++ [4] results in [3,2,1,4]. The application of the reverse function binds very tightly, so we do it first, then concatenate.
- reverse \$ [1,2,3] ++ [4] results in [4,3,2,1]. We concatenate first, then apply the reversing function. Same as reverse ([1,2,3] ++ [4]).

Practical

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# **Function Composition**

We used *function composition* to combine our functions together. The mathematical  $(f \circ g)(x)$  is written (f . g) x in Haskell.

In Haskell, operators like function composition are themselves functions. You can define your own!

-- Vector addition (.+) :: (Int, Int) -> (Int, Int) -> (Int, Int) (x1, y1) .+ (x2, y2) = (x1 + x2, y1 + y2)

(2,3) .+ (1,1) == (3,4)

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# **Function Composition**

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In Haskell, operators like function composition are themselves functions. You can define your own!

(2,3) .+ (1,1) == (3,4)

You could even have defined function composition yourself if it didn't already exist:

(.) :: 
$$(b \rightarrow c) \rightarrow (a \rightarrow b) \rightarrow (a \rightarrow c)$$
  
(f . g) x = f (g x)

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## Conditionals

Demo: polarity using guards, if statements.

Demo: (if we have time), loops via recursion.

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## Lists

We used a bunch of list functions. How could we implement them ourselves??

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# Lists

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Lists are singly-linked lists in Haskell. The empty list is written as [] and a list node is written as x : xs. The value x is called the head and the rest of the list xs is called the tail. Thus:

"hi!" == ['h', 'i', '!'] == 'h':('i':('!':[])) == 'h' : 'i' : '!' : []

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# Lists

We used a bunch of list functions. How could we implement them ourselves??

Lists are singly-linked lists in Haskell. The empty list is written as [] and a list node is written as x : xs. The value x is called the head and the rest of the list xs is called the tail. Thus:

"hi!" == ['h', 'i', '!'] == 'h':('i':('!':[])) == 'h' : 'i' : '!' : []

When we define recursive functions on lists, we use the last form for pattern matching:

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# **Equational Evaluation**

map f [] = []

map f (x:xs) = f x : map f xs

We can evaluate programs *equationally*:

```
map toUpper "hi!"
```

Practical

# **Equational Evaluation**

map f [] = []

map f (x:xs) = f x : map f xs

We can evaluate programs *equationally*:

map toUpper "hi!" = map toUpper ('h':"i!")



Practical

# **Equational Evaluation**

map f [] = []

map f (x:xs) = f x : map f xs

We can evaluate programs *equationally*:

map toUpper "hi!"  $\equiv$  map toUpper ('h':"i!")  $\equiv$  toUpper 'h' : map toUpper "i!"

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map toUpper "hi!"  $\equiv$  map toUpper ('h':"i!") ≡ toUpper 'h' : map toUpper "i!"  $\equiv$  'H' : map toUpper "i!"  $\equiv$  'H' : map toUpper ('i':"!") ∃ 'H' : toUpper 'i' : map toUpper "!"

Practical 0000000

# **Equational Evaluation**

map f [] = []map f (x:xs) = f x : map f xs

We can evaluate programs *equationally*:

map toUpper "hi!"  $\equiv$  map toUpper ('h':"i!") = toUpper 'h' : map toUpper "i!"  $\equiv$  'H' : map toUpper "i!"  $\equiv$  'H' : map toUpper ('i':"!") ∃ 'H' : toUpper 'i' : map toUpper "!"  $\equiv$  'H' : 'I' : map toUpper "!"

Practical 0000000

# Equational Evaluation

map f [] = []map f (x:xs) = f x : map f xs

We can evaluate programs *equationally*:

map toUpper "hi!"  $\equiv$  map toUpper ('h':"i!") = toUpper 'h' : map toUpper "i!"  $\equiv$  'H' : map toUpper "i!"  $\equiv$  'H' : map toUpper ('i':"!") 'H' : toUpper 'i' : map toUpper "!"  $\equiv$  $\equiv$  'H' : 'I' : map toUpper "!"  $\equiv$  'H' : 'I' : map toUpper ('!':"")

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# **Equational Evaluation**

map f [] = []
map f (x:xs) = f x : map f xs

We can evaluate programs *equationally*:

map toUpper "hi!"

$\equiv$	$\mathtt{map}$	toUpp	er ('h':"i!")	
$\equiv$	toUp	per '	h': map toUpper "i!"	
$\equiv$	'H'	: map	toUpper "i!"	
$\equiv$	'H'	: map	toUpper ('i':"!")	
$\equiv$	'H'	: tol	pper 'i' : map toUpper	"!"
$\equiv$	'H'	: 'I'	: map toUpper "!"	
$\equiv$	'H'	: 'I'	: map toUpper ('!':""	)
$\equiv$	'H'	: 'I'	: '!' : map toUpper	

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# **Equational Evaluation**

map f [] = []
map f (x:xs) = f x : map f xs

We can evaluate programs *equationally*:

map toUpper "hi!"

$\equiv$	$\mathtt{map}$	to	oUppe	er	('h	':"i	!")		
$\equiv$	toUp	pe	er 'h	ı'	: r	nap	toUpp	per "i!"	
$\equiv$	'H'	:	$\mathtt{map}$	t	oUppe	er "	i!"		
$\equiv$	'H'	:	$\mathtt{map}$	t	oUppe	er (	'i':'	'!")	
$\equiv$	'H'	:	toUp	ope	er 'i	i':	$\mathtt{map}$	toUpper	" ! "
$\equiv$	'H'	:	'I'	:	$\mathtt{map}$	toU	pper	"i"	
$\equiv$	'H'	:	'I'	:	$\mathtt{map}$	toU	pper	('!':"")	
$\equiv$	'H'	:	'I'	:	'!'	:	$\mathtt{map}$	toUpper	
$\equiv$	'H'	:	'I'	:	'!'	:	$\mathtt{map}$	toUpper	[]
Haskell

Practical

## **Equational Evaluation**

map f [] = []
map f (x:xs) = f x : map f xs

We can evaluate programs *equationally*:

map toUpper "hi!"

$\equiv$	$\mathtt{map}$	t	oUppe	er	('h'	':"i	!")		
$\equiv$	toUp	ppe	er 'h	ı'	: n	nap	toUpp	per "i!"	
$\equiv$	'H'	:	$\mathtt{map}$	t	oUppe	er "	i!"		
$\equiv$	'H'	:	$\mathtt{map}$	t	oUppe	er (	'i':'	'!")	
$\equiv$	'H'	:	toUp	ppe	er 'i	i':	$\mathtt{map}$	toUpper	" ! "
$\equiv$	'H'	:	'I'	:	$\mathtt{map}$	toU	pper	"!"	
$\equiv$	'H'	:	'I'	:	$\mathtt{map}$	toU	pper	('!':"")	)
$\equiv$	'H'	:	'I'	:	, i ,	:	$\mathtt{map}$	toUpper	
$\equiv$	'H'	:	'I'	:	, i ,	:	$\mathtt{map}$	toUpper	[]
$\equiv$	'Н'	:	'I'	:	, i,	:	[]		

Haskell

Practical

## **Equational Evaluation**

map f [] = []
map f (x:xs) = f x : map f xs

We can evaluate programs *equationally*:

map toUpper "hi!"

$\equiv$	map t	oUppe	er	('h	':"i	!")		
$\equiv$	toUpp	er 'h	ı'	: n	nap	toUpp	per "i!"	
$\equiv$	'H' :	$\mathtt{map}$	to	Uppe	er "	i!"		
$\equiv$	'H' :	$\mathtt{map}$	to	Uppe	er (	'i':'	'!")	
$\equiv$	'H' :	toUp	ppe	r'i	i':	$\mathtt{map}$	toUpper	" ! "
$\equiv$	'H' :	'I'	:	map	toU	pper	"i"	
$\equiv$	'H' :	'I'	:	map	toU	pper	('!':"")	
$\equiv$	'H' :	'I'	:	'!'	:	$\mathtt{map}$	toUpper	
$\equiv$	'H' :	'I'	:	'!'	:	$\mathtt{map}$	toUpper	[]
$\equiv$	'H' :	'I'	:	'!'	:	[]		
=	"HT!"							

Haskell





The quiz will be up on the course website sometime on Thursday.

## Warning

The quiz is assessed. The deadline is the end of next Thursday.

